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System Identification Technology for Estimating Re-entry Vehicle Aerodynamic Coefficients

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This paper describes the development and application of a systematic system identification technology to obtain accurate estimates of maneuvering re-entry vehicle aerodynamic coefficients from flight data. The system identification procedure is divided into two phases, preflight system evaluation and postflight data processing. Preflight evaluation produces test modifications which are likely to maximize estimation accuracy, within mission constraints. Postflight processing obtains aerodynamic models and parameter estimates with minimum error from flight data. The impact of the specific maneuvering re-entry vehicle characteristics on system identification are described and appropriate solutions are proposed. Examples illustrate the effectiveness of the technology.

Introduction

THE demonstration of the potential contributions of maneuvering reentry vehicle technology to the accuracy of strategic missile systems is a principal objective of a current maneuvering re-entry vehicle (MaRV) program. This advanced technology program is a significant element in the evolution of a missile system capable of responding to a variety of strategic requirements. The MaRV flight test results constitute principal milestones of this program.

Because of the limited number of flights to evaluate performance of MaRV vehicles, the incremental contribution of each test flight to the vehicle data base must be maximized in order to validate vehicle design and extrapolate results to future system improvements in the most cost-effective manner. One critical element of the vehicle performance data base is the set of aerodynamic characteristics, which is fundamental to the design of accurate guidance and control systems for the vehicle. The aerodynamic data may be summarized in a vehicle nonlinear digital simulation. This simulation is used to predict vehicle responses under a range of environmental and vehicular based error-inducing factors, which must be compensated for by the guidance and control inputs. Thus, the aerodynamic data base is critical to improved system accuracy.

This paper presents the status of a computer-aided methodology which has previously been demonstrated to offer significant capability for quantifying aircraft and missile aerodynamic characteristics.[1] This methodology, system identification, has further been demonstrated to provide an evaluation of systematic error sources in instrumentation and postflight data processing software.[2] The following sections discuss specialization of the methodology to future maneuvering vehicle tests and development of preliminary planning considerations which enhance the quality of the data base extracted from the tests.

Requirements and Overview of System Identification Approach

The dynamics and aerodynamics of a maneuvering re-entry vehicle (MaRV) flights are affected by many complex factors which are difficult to predict analytically or from tunnel tests. Aerodynamic characteristics of such vehicles may be obtained accurately only from flight data. The estimation accuracy is improved by careful preflight analysis coupled with specialized techniques for postflight data processing. The following MaRV characteristics dictate specific requirements for system identification techniques:

1) The flight trajectory of a MaRV may pass through extreme regions of high angle-of-attack, Mach number, and Reynolds number. In general, the aerodynamic characteristics may depend on angle-of-attack, control deflection, Mach number, and Reynolds number in an unknown way.

2) The control effectiveness is nonlinear and depends significantly on other aerodynamic variables. In addition, certain regions are subject to high control deflection accelerations, leading to significant inertia reaction forces, commonly referred to as tail-wags-dog (TWD) phenomenon.

3) Important aerodynamic variables, such as angle-of-attack and sideslip angle, cannot be measured.

4) A significant portion of the control deflection occurs in response to the feedback control system. Such inputs sometimes cause problems in aerodynamic coefficient estimation.

5) The open-loop MaRV dynamics are unstable under some flight conditions. The identification methods for MaRV data processing must be capable of handling unstable systems when it is desired to identify the open-loop vehicle characteristics from closed-loop data.

These considerations affect both the preflight system evaluation and postflight data processing. In preflight evaluation, model accuracy requirements are compared with expected estimation errors from a specific test. This leads to specifications of necessary test modifications to meet accuracy requirements, within the constraints of the overall mission. The postflight data processing extracts the most accurate model from noisy flight data, making the best use of a priori information (e.g. from wind tunnels or previous flight tests). These steps are now described in detail.

Preflight Analysis

Prelight analysis evaluates the effects of the following three factors on parameter estimation accuracy:

1) *Instrument errors and failures.* This may also include specification of a minimal set of instruments, which must be available to provide acceptable parameter estimation accuracy.

2) *Sampling rate, wordlength* and other telemetry parameters.

3) *Trajectories* and acceptable trajectory perturbations which enhance estimation accuracy.

The analysis is based on a detailed simulation of the MaRV and uses statistical techniques for accuracy evaluations. The

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Table 1 Effect of systematic instrument errors on aerodynamic coefficient estimation accuracy

Estimation approach to handle systematic instrument errors	Error even if systematic errors were absent	Additional error because of systematic errors
Systematic errors not identified, set to calibration values	M_{11}^{-1}	$M_{11}^{-1}M_{12}(\phi_c-\phi)(\phi_c-\phi)^TM_{21}M_{11}^{-1}$
Systematic errors identified concurrently with aerodynamic variables	M_{11}^{-1}	$M_{11}^{-1}M_{12}(M_{22}-M_{21}M_{11}^{-1}M_{12})^{-1}M_{21}M_{11}^{-1}$

simulation must emulate the behavior of the real vehicle as closely as possible. In this application the General Trajectory Simulation (GTS),[3] a sophisticated and accurate simulation of MaRV flight phenomenon, is used.

Instrumentation Error Analysis

MaRV on-board and ground-based instruments are subject to a variety of error sources. For the purpose of evaluating their effect on parameter estimation accuracy, instrument errors can be divided in two broad classes called *systematic errors* and *random errors*. *Systematic errors* are deterministic errors which remain constant or nearly constant during re-entry. Examples of these errors are bias, scale factor, and quadratic and higher-order nonlinearities. Accurate calibrations just prior to the test can eliminate these errors or minimize their effect. These errors may also be estimated concurrently with aerodynamic parameter values. This simultaneous estimation apportions the information in a finite amount of data over more parameters and so may or may not help improve estimation accuracy of aerodynamic parameters. The *random errors* are basically different. As the name implies, their randomness allows estimation of only their associated statistical characteristics. They produce random variations in parameter estimates. Random errors may generally be approximated to be white and Gaussian so that they may be specified by mean and covariance. The effects of these two types of errors on parameter estimation accuracy are described in Ref. 2.

Let

θ = $m \times l$ vector of aerodynamic coefficients of interest (e.g., C_{m_α}, $C_{m_{\alpha\delta}}$)

ϕ = $m' \times l$ vector of systematic errors associated with instruments (e.g., accelerometer bias, gyro misalignment)

ϕ_c = preflight calibration estimates of instrument characteristics

$J(\theta,\phi,\phi_c)$ = negative log likelihood function of the parameters given the measurements‡

$$M=\begin{bmatrix}\partial^2J/\partial\theta^2 & \partial^2J/\partial\theta\partial\phi\\ \partial^2J/\partial\phi\partial\theta & \partial^2J/\partial\phi^2\end{bmatrix}=\begin{bmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{bmatrix}\tag{1}$$

(the information matrix)

Table 1 summarizes parameter estimation errors for two different methods of handling the systematic errors.[2] The choice between the two methods for treating the systematic errors depends on the relative magnitude of the second term.

To treat random errors, the noise in any instrument is assumed uncorrelated with the noise in any other instrument. Then, the information matrix for parameters, θ, is

$$M=\sum_{i=1}^{p}\frac{M_{ii}}{r_{ii}}\tag{2}$$

where

$$M_{ii}=\int_0^T\frac{\partial h_i^T}{\partial\theta}\frac{\partial h_i}{\partial\theta}\mathrm{d}t\tag{3}$$

and r_{ii} is the intensity of the noise in the ith instrument. The information matrix, M, is, therefore, a sum of p terms, each term representing the contribution of one instrument towards the total information matrix. The contribution of any instrument is inversely proportional to the mean square random error in the instrument. These results may be extended when the instrument errors are correlated.

Telemetry Requirements

Telemetry requirements are major considerations in maneuvering re-entry vehicle testing. Two elements of output measurements affect telemetry bandwidth – the sampling rate and the wordlength. The extent to which these factors affect the ultimate parameter estimates depends upon the trajectory, the frequency content of the control input, and the kinds of dynamic instruments, together with their bandwidths.

When the wordlength is relatively long, such that there are significant variations in discrete outputs from one sample point to the next, the effect of discretization may be approximated by uncorrelated noise in the output. If the discretization interval is Δ, this noise is uniformly distributed between $\pm\Delta/2$ (assuming roundoff; if truncation is carried out, the error is uniformly distributed between 0 and Δ). For the purpose of our analysis, we assume that the noise is normally distributed, with the same rms value, $\Delta/\sqrt{12}$, as the uniformly distributed noise. For short word-lengths where the discrete output does not change at all, or changes by a few bits between two consecutive sample points, the following considerations also become important: 1) differentiation and integration accuracies, 2) random error equivalence of

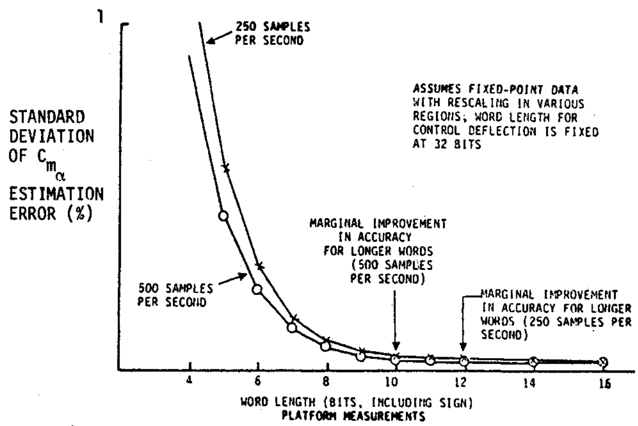

Fig. 1 Effect of discretization wordlength and sample rate on estimation accuracy of C_{m_α}.

‡The likelihood function is an extremely important concept in statistical estimation theory. The likelihood function of a parameter based on a set of measurements is closely related to the conditional probability density function of the measurements given the parameter. For further discussion of the likelihood function, see Ref. 4. The information matrix is the second gradient of the negative log-likelihood function with respect to parameters.

truncation error, 3) random error in measurements, 4) scaling, 5) range of variations of outputs, and 6) model complexity.

The major consideration in determining the minimum acceptable *sampling rate* is the necessary bandwidth in the measurement. This is determined by the nature of the trajectory and the aerodynamic characteristics of the vehicle. The sampling rate is related to the required bandwidth via the Nyquist rate and should be at least 5 to 10 times the highest frequency of interest in practical cases. In general, the following factors dictate selection of sampling rates: 1) channel reconstruction (for example, computation of second time derivative of control surface deflection in the determination of inertia forces), 2) rate of change of variables being sampled (the sampled quantity should change by a few significant figures between consecutive samples), 3) noise averaging, 4) modeling errors and process noise, and 5) model complexity.

Trajectory Design

Methods for the selection of inputs which maximize parameter accuracy, are discussed in Refs. 5 and 6. The techniques presented in the references are specifically applicable to linear systems and the only constraints considered are quadratic functions of state variables and inputs.

The trajectory of a maneuvering re-entry vehicle is usually defined based on considerations more important than system identification. Therefore, any trajectory shaping technique must only produce small deviations in the trajectory. Nevertheless, significant control activity produced by the high-gain feedback control system causes a considerable excitation of the vehicle, improving parameter identifiability. Therefore, application of additional perturbing inputs may be unnecessary, but should be considered in the test design phase.

Under the above constraints, the algorithm for computing trajectory perturbations, to enhance parameter identifiability, is as follows:

1) Linearize the simulation equation about the nominal trajectory.

2) Select a reasonable perturbation level.

3) Design an input for the linearized system using the techniques of Ref. 5.

4) Simulate the input to ensure that the perturbation does not affect the trajectory significantly.

Preflight Evaluation of Effects of Telemetry Parameters on Estimation Errors

To demonstrate the procedure for evaluating the effect of telemetry variables on parameter estimation accuracy, we consider measurements from a four axis inertial platform with a triad of accelerometers and gyros. Figure 1 shows the fractional error in the estimate of the slope of pitch moment vs. angle-of-attack curve, C_{m_α}, as a function of the measurement sampling rate and wordlength. Fixed point data with optimal scaling is assumed.§

Figure 1 shows how small wordlength may tremendously increase estimation error on C_{m_α}. There are two curves for 250 samples per second and 500 samples per second. Wordlength beyond 12 bits provides a marginal improvement in accuracy at 250 samples per second. At 500 samples per second the necessary wordlength reduces to 10 bits. The degradation in accuracy is serious below 8 bits.

Postflight Analysis

Figure 2 gives the functional flow chart of the postflight data processing technique for identifying MaRV aerodynamic models. A significant issue in application of system identification to postflight data processing is the degree to which the following steps are simultaneously performed: 1) *reconstruction of system states,* including digital filtering of the data, estimation of unmeasured states, and estimation of aerodynamic force and moment time histories by open-loop integration; 2) *Model Structure Determination (MSD)* by regression and hypothesis testing methods; and 3) *parameter identification* of the aerodynamic models using advanced algorithms which compute, in addition to parameters of aerodynamic derivatives, their uncertainties, and the sensor errors and their uncertainties.

Data Reconstruction

Data reconstruction is the process of computing, from the measurements, the unmeasured quantities necessary for the identification stage. In general, it produces approximate estimates of instrument errors and a smoothed estimate of the trajectory. Either concurrently or in a second stage, the aerodynamic coefficients and variables, such as pitch and yaw angles-of-attack, are computed with appropriate kinematic relationships.

The reconstruction for maneuvering re-entry vehicle model identification is based on both the on-board and the ground-based data. On-board dynamic measurements useful for identification are the three-axis body angular rates, the three linear accelerations, and control deflections. Ground-based measurements include radars and optical cameras from which smoothed estimates of altitude, total velocity, and dynamic pressure are obtained using techniques of trajectory estimation. The following time histories are of interest: 1) pitch angles-of-attack and sideslip angle, 2) three-axis moment coefficient time histories, 3) three-axis force coefficient time histories, 4) control acceleration, and 5) Euler angles, required to incorporate gravity effects and to correlate on-board and ground-based data.

The MaRV equations of motion in the body axis system are

$$\dot{V}_b = -\omega \times V_b + a_{cg} + g \qquad (4)$$

$$\dot{\omega} = I^{-1}\left[q_\infty S b \begin{bmatrix} C_l \\ C_m \\ C_n \end{bmatrix} + \begin{bmatrix} \mathrm{TWD}_p \\ \mathrm{TWD}_q \\ \mathrm{TDW}_r \end{bmatrix} - (\omega \times I)\omega \right] \qquad (5)$$

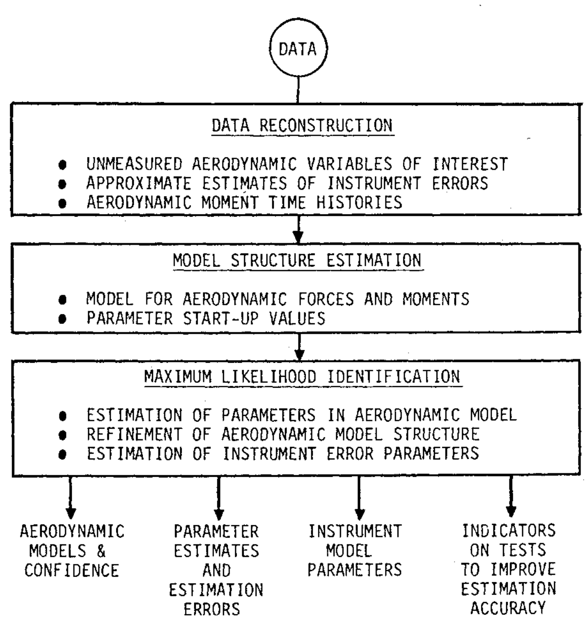

Fig. 2 Postflight-data processing techniques for maneuvering re-entry vehicle aerodynamic model estimation.

§Fixed point arithmatic is assumed because of hardware and speed. Constraints multiple scaling has been used occasionally with improved results. Floating point arithmatic will reduce error and provide an improved tradeoff between instrument errors and roundoff errors.

$$\dot{\Phi} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi/\cos\theta & \cos\phi/\sin\theta \end{bmatrix} \omega \qquad (6)$$

where

$V_b = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$ = components of vehicle speed in body axis system

$\omega = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$ = components of vehicle angular rates in body axis system

$\Phi = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}$ = Euler angle

$a_{cg} = \begin{bmatrix} a_{cg_x} \\ a_{cg_y} \\ a_{cg_z} \end{bmatrix}$ = components of vehicle linear acceleration in body axis system

$g = \begin{bmatrix} g\sin\theta \\ g\sin\phi\cos\theta \\ g\cos\phi\cos\theta \end{bmatrix}$ = components of gravity in body axis system

$I = \begin{bmatrix} I_x & I_{xy} & I_{xz} \\ I_{xy} & I_y & I_{yz} \\ I_{xz} & I_{yz} & I_z \end{bmatrix}$ = moment of inertia matrix

C_l, C_m, C_n = roll, pitch and yaw moment coefficients

TWD_p TWD_q, TWD_r = roll, pitch and yaw components of tail-wags-dog inertial reaction moment

q_∞ = freestream dynamic pressure
S = reference surface area
b = reference length

The above equations assume that the vehicle is rigid. The last equation is simply the kinetic relationship for body orientation. Noisy measurements of a_{cg} and ω are available. Tail-wags-dog (TWD) forces and moments may be computed from measurement of control deflection.

Reconstruction of Angle-of-Attack and Sideslip Angle

A direct method to obtain the velocity and orientation variables is to substitute measured values for ω and a_{cg} in the above equations, and to integrate Eqs. (4) and (6) to obtain V_b and Φ. The angle-of-attack α and sideslip angle β are obtained from velocity components in the body axis system

$$\alpha = \arcsin(w/V) \qquad \beta = \arcsin(v/V) \qquad V = \sqrt{u^2+v^2+w^2}$$

There is one major problem with this approach. Equation (4) is neutrally stable; therefore, estimates of linear velocities are sensitive to the initial conditions and measurement errors. This affects estimates of angle-of-attack and sideslip angles, which are derived directly from the linear velocities. There are two ways of solving this stability problem in Eq. (4): 1) use trajectory data or 2) use a model for aerodynamic force coefficients.

Though the ground-based data cannot measure vehicle orientation, it can predict vehicle trajectory. Equation (6) may then be integrated open-loop to compute the orientation (these equations are stable). Relative body orientation with respect to the trajectory, then, gives angle-of-attack and sideslip angle. Equation (5) is then solved to obtain the aerodynamic moment coefficient time histories.

The force coefficient model method utilizes the fact that the model for the aerodynamic force coefficients is often quite simple. The normal force coefficient, for example, may usually be written as a linear or quadratic function of the angle-of-attack and linear function of the control deflection. Suitable models are used also for the side force and axial force coefficients. Therefore, the measured acceleration is a sum of two terms, one of which is a known function of the velocity components and flap deflection, and the second term is the inertial tail-wags-dog force which may be computed. Equation (4) is then written only in terms of the body velocity components and the accelerations are considered as measurements. This is used to set up a filtering formulation; thus

$$a_{cg} = f'(\alpha,\beta,V,\delta) \;\triangleq\; f(V_b,\delta) + \mathrm{TWD}_f \qquad (9)$$

$$\dot{V}_b = -\omega \times V_{b'} + f(V_b,\delta) + g + \mathrm{TWD}_f \qquad (10)$$

where TWD is the tail-wags-dog intertial force. The measured value of the acceleration is noisy

$$(a_{cg})_{\text{meas.}} = f(V_b,\delta) + \mathrm{TWD}_f + \text{noise} \qquad (11)$$

In this Kalman filtering formulation, Eq. (10) is the state equation and Eq. (11) is the measurement equation. Angle-of-attack and sideslip angle are obtained from filtered velocity components.

Data Reconstruction for Tail-Wags-Dog Modeling

The structural vibration dynamics are not considered in this study since the lowest body-bending mode frequency is sufficiently above the TWD oscillations. Thus, all the necessary dynamic effects are included by filtering the measurements at the appropriate frequencies and modeling the TWD.

Neglecting the cross product inertial terms, the pitching moment coefficient equation is

$$C_m = \frac{1}{q_\infty Sb}\{I_y\dot{q} - (I_z - I_x)pr + [\ddot{\delta}(I_f - m_f rd\cos\delta)]\} \qquad (12)$$

where

I_f = the flap inertia about the hinge line
m_f = the flap mass
d = distance from hinge line to flap c.g.
r = distance from vehicle c.g. to hinge line along body x-axis
δ = flap angular deflection
$\ddot{\delta}$ = flap angular acceleration

$\cos\delta$ is approximately equal to one. The pitching moment is therefore a linear combination of three terms, two of which must be obtained by differentiating signals in the reconstruction step. Practical aspects of differentiating noisy signals are extremely important in reentry vehicle parameter identification. These are discussed in the following.

The significance of the TWD term in the simulation can be seen in Figs. 3 and 4. Figure 3 reconstructs C_m neglecting TWD while Fig. 4 includes this important effect. The region shown includes two effects observed on MaRV, a high-frequency kinematic limit cycle and a low-frequency aerodynamic limit cycle. (Both of these limit cycles result from high-gain feedback from the autopilot.) Note that the TWD term is relatively more important in the kinematic limit cycle regime.

Estimation of Moment and Force Coefficients

The force coefficients are obtained directly from the measured acceleration transferred to vehicle c.g. (after subtracting the TWD force). Equation (5) gives the moment coefficients. These require the differentiation of the angular rates and double differentiation of the control deflection (to

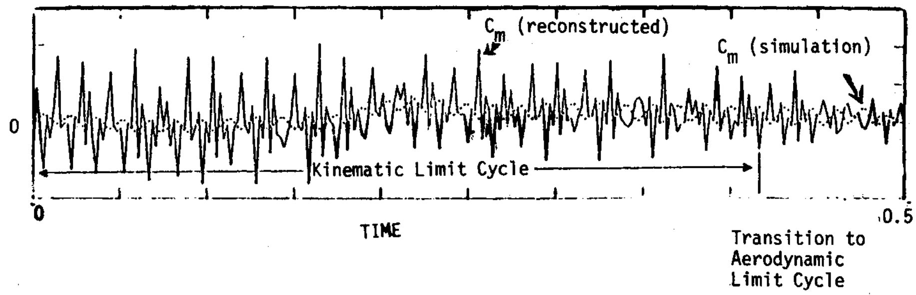

Fig. 3 Reconstructed pitching moment neglecting tail-wags-dog.

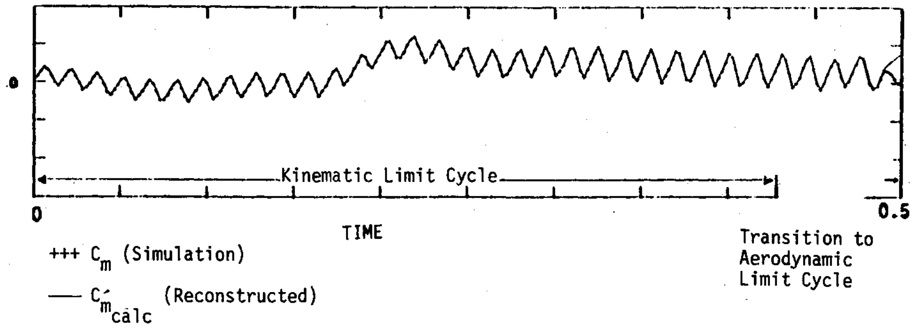

Fig. 4 Reconstructed pitching moment including tail-wags-dog.

compute its acceleration for the TWD terrms). In the computation of force coefficients, the off c.g. and TWD effects are small; therefore, accurate differentiation of angular rates and control deflection is not so critical. Both terms are important in the moment equation, however. Therefore, finding the first derivatives of p, q, r and the second derivative of δ accurately is critical for moment coefficient reconstruction.

Techniques for Differentiation of Signals During Reconstruction

Direct differentiation of signals contaminated with noise usually gives results with even a higher noise level. The quality of the signal gets poorer as the order of the differentiation is increased. There are several methods to control the error and maintain reasonable signal-to-noise ratio:

Method 1: To find the derivative at any point on the trajectory, *a polynominal is fitted to a number of data points around the point of interest using least-squares.* The polynomial should have fewer parameters than the number of data points considered. The fitted polynomial is differentiated to produce the desired derivative. The method is time-consuming and requires a priori selection of polynomial order and number of data points to which the polynomial is fitted.

Method 2: A spline function (i.e. piecewise polynomial with certain continuity conditions) with variable or fixed knots (i.e. junction points) is fitted to the entire time history. The spline function is differentiated to obtain derivatives. This method is better than the previous one, but suffers from some of the same problems.

Method 3: The time history is Fourier transformed. The Fourier transform is multiplied by $(j\omega)^n$ if we are interested in the nth derivative of the function. The coefficients above the highest frequency of interest are dropped. An inverse Fourier transform, then, gives the nth derivative of the function (ω is the frequency).

Because of the availability of the fast Fourier transform techniques, the third method is extremely efficient and only requires the selection of a highest frequency of interest. This is the method developed for differentiation of signals in this MaRV data processing.

Model Structure Development

The Model Structure Development (MSD) is an extremely important stage in the overall system identification approach for MaRV flight data processing. A good model, at the minimum, would 1) give insight into the applicable physical phenomena, 2) explain the force and moment coefficients on the trajectory, and 3) have a capability to predict the nonlinear aerodynamics in the neighborhood of the flight trajectory.

In general, the model structure determination problem is to explain the behavior of aerodynamic coefficient time histories in terms of independent aerodynamic variables (e.g. angle-of-attack, sideslip angle, control deflection, Reynolds number, and Mach number) or functions of these variables.

In general, the model structure determination may be divided into three distinct subproblems: 1) selection of general functional forms to relate the moments and forces to independent variables for use with noisy test data, 2) criteria to compare competing models in order to determine the model with the best predictive and other capabilities for a class of response, and 3) efficient computational techniques for determining the models from test data. The details of the model structure determination process are given in Ref. 7.

Let C be an aerodynamic coefficient which is known to be related to n independent variables $z_1, z_2, \ldots, z_n$, but the form of the relationship is not known. A general polynomial representation for C is

$$C = \sum_{j_1=0}^{m_1} \sum_{j_2=0}^{m_2}, \ldots, \sum_{j_n=0}^{m_n} C_{j_1,j_2,\ldots,j_n} z_1^{j_1} z_2^{j_2}, \ldots z_n^{j_n} \tag{13}$$

This representation for the aerodynamic coefficient requires selection of the highest powers in the expansion $m_1, m_2, \ldots, m_n$.

A general polynomial in pitch angle-of-attack, α, control deflection, δ, and Mach number (or Reynolds number), μ, to model pitching moment coefficient is (a similar polynomial is used for normal force coefficient):

$$\begin{aligned} C_m = {} & C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\alpha^2}}\alpha^2 + C_{m_{\alpha^3}}\alpha^3 + C_{m_{\alpha^4}}\alpha^4 \\ & + C_{m_{\alpha\delta}}\alpha\delta + C_{m_{\alpha\delta^2}}\alpha\delta^2 + C_{m_{\alpha\delta^3}}\alpha\delta^3 + C_{m_{\alpha^2\delta}}\alpha^2\delta \\ & + C_{m_{\alpha^2\delta^2}}\alpha^2\delta^2 + C_{m_\delta}\delta + C_{m_{\delta^2}}\delta^2 + C_{m_{\delta^3}}\delta^3 + C_{m_{\delta^4}}\delta^4 \\ & + C_{m_{\alpha^2\delta^3}}\alpha^2\delta^3 + C_{m_{\alpha^3\delta}}\alpha^3\delta + C_{m_{\alpha^3\delta^2}}\alpha^3\delta^2 + C_{m_{\alpha^3\delta^3}}\alpha^3\delta^3 \\ & + C_{m_\mu}\mu + C_{m_{\mu^2}}\mu^2 + C_{m_{\mu^3}}\mu^3 + C_{m_{\alpha\mu}}\alpha\mu + C_{m_{\alpha^2\mu}}\alpha^2\mu \\ & + C_{m_{\alpha^3\mu}}\alpha^3\mu + C_{m_{\delta\mu}}\delta\mu + C_{m_{\delta^2\mu}}\delta^2\mu + C_{m_{\delta^3\mu}}\delta^3\mu + C_{m_q}q \end{aligned} \tag{14}$$

The above expansion may be performed about trim values of α, δ, μ, and q, if desired. Higher-order expansion terms should usually be avoided in the aerodynamic models. When the data region is so big and diverse that high-order polynomials are required to represent the aerodynamics accurately, polynomial spline representations must be used.

The advantage of spline functions is that they allow the aerodynamics to change as the trajectory moves from one region to the next while retaining a low-order polynomial representation. A spline function[8] $S_{m,\nu}(x)$ may be used to represent the true aerodynamic coefficient in terms of a single independent variable, x, as follows:

$$C_{\text{aero}}(x) \approx S_{m,\nu}(x) = \sum_{j=0}^{m} C_{1j}x^j + \sum_{l=2}^{k} \sum_{j=\nu+1}^{m} C_{lj}(x-x_l)^j_+ \quad (15)$$

where x^j_+ is x^j for $x \geq 0$ and is zero otherwise. The spline function is a different polynomial in different regions, e.g.

$$C(x) = \sum_{j=0}^{m} C_{1j}x^j \qquad (x<x_2)$$

$$C(x) = \sum_{j=0}^{m} C_{1j}x^j + \sum_{j=\nu+1}^{m} C_{2j}(x-x_2)^j_+ \qquad (x_2<x<x_3) \quad (16)$$

Note that the function is continuous up to order ν because of the form selected for the additive term.

This representation can be extended when there is more than one independent variable. The specification of the approximating spline requires the selection of four variables: 1) polynomial order m and order of continuity ν, 2) number and position of knots, 3) specific terms which could be deleted, and 4) coefficients C_{ij}. The spline representation is linear in unknown parameters C_{ij}.

The functional form characteristics must be now integrated with techniques for specifying criteria of model acceptability and computation techniques. This integration is detailed in Ref. 7.

Maximum Likelihood Technique for Parameter Identification

The maximum likelihood method has been described previously.[1] There are several problems with the application of the standard technique to MaRV aerodynamic coefficient estimation. Some of the problems are:

1) The open-loop MaRV equations of motion are unstable and the feedback control law is not modeled. Therefore, when the parameters are not at the correct values, the computed trajectory tends to diverge.

2) The extended Kalman filter for MaRV equations of motion is computationally infeasible.

3) Computationally efficient methods to optimize the likelihood function for MaRV test data are required.

The first two problems are more important and are now described.

Stabilization Techniques for Nonlinear Systems

The minimization of the negative log-likelihood function (NLLF) with respect to the parameters requires integrating the equations of motion [Eqs. (4-6) and Eq. (10)]. MaRV open-loop equations of motion are unstable, but the closed-loop equations are stable. Therefore, if the feedback control law were known exactly, the closed-loop equations of motion could be propagated. Since, in general, the control law may not be available, the open-loop equations must be used. These equations are highly unstable if the parameters are different from the true values.

Suitable filters are required to maintain trajectory stability in the early phase of the iterative numerical procedure. Three such filters are as follows:

1) *Simplified Extended Kalman Filters:* Add an artificial noise to the state equations and measurement equations if necessary. Use a suboptimal Kalman filter based on steady-state gains with gains updated at regular intervals. Several other simplifications are also possible. The approach is quite complex, but is implementable and works well when the parameters are close to the true values. The disadvantages are that the process and measurement noise statistics must be chosen a priori and equations for derivatives of NLLF are still quite complex.

2) *Constant Gain Filter:* Use a pseudofilter, possibly time-varying, such that the equations are adequately stable; i.e., select gains K_a such that

$$\dot{\hat{x}} = f(\hat{x},u,\theta,t) + K_a[y-h(\hat{x},u,\theta,t)] \quad (17)$$

forms a stable system. The advantages are that: a) it is a simple approach; b) filter gains are chosen directly, therefore, the process noise statistics need not be selected; and c) minimum measurement noise need be fed through the equations.

3) *State Bounds:* Use a priori bounds on the deviations of computed state variables from measured and derived value and all the associated sensitivities are set to zero. The approach is simple and does not feed the noise in the equations. The convergence is slow under certain circumstances.

Filters 2 and 3 above have been most often used with success. These filters are eliminated as the parameters converge to the true values.

Evaluation of Identified Coefficient Accuracy on Simulated Data

Application of the identification methodology to simulated responses is useful for evaluating accuracy of the computational algorithms. This is because the aerodynamic forces and moments are known from the simulation data base, and hence direct comparisons of estimated results may be effected.

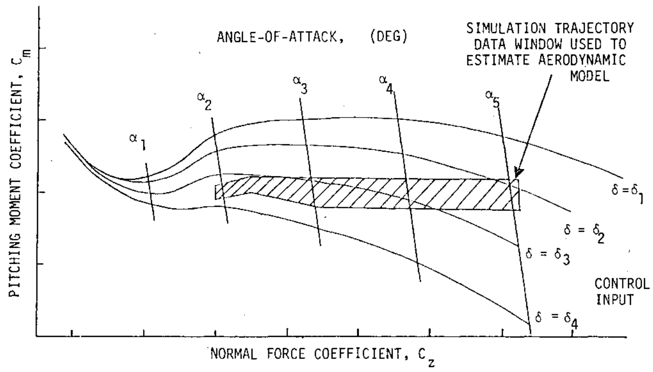

Fig. 5 C_m vs C_z characteristics used in simulation.

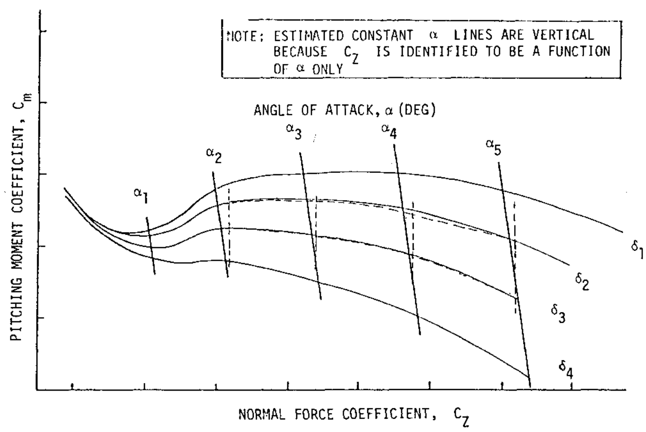

Fig. 6 Estimated and simulation C_m vs C_z characteristics over the range of variables used in identification.

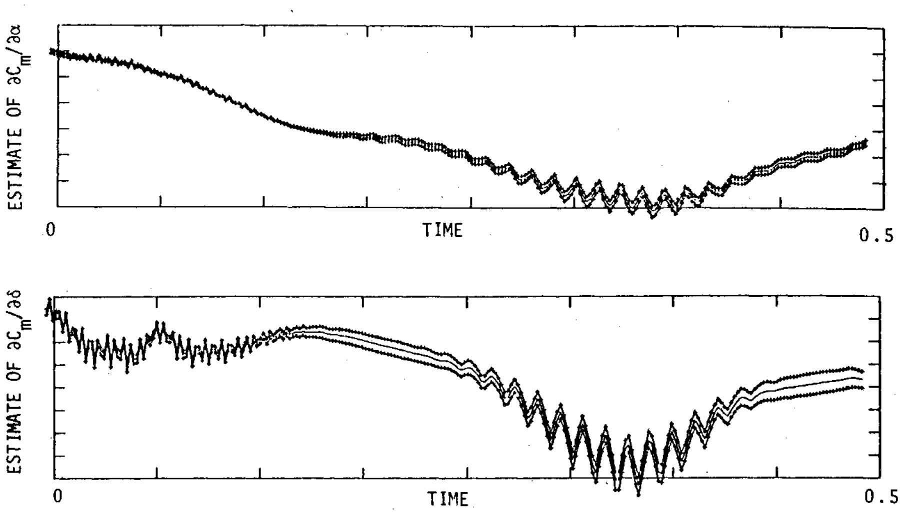

Fig. 7 Estimated M_α and M_δ with 2σ bound (TWD inertia forces included).

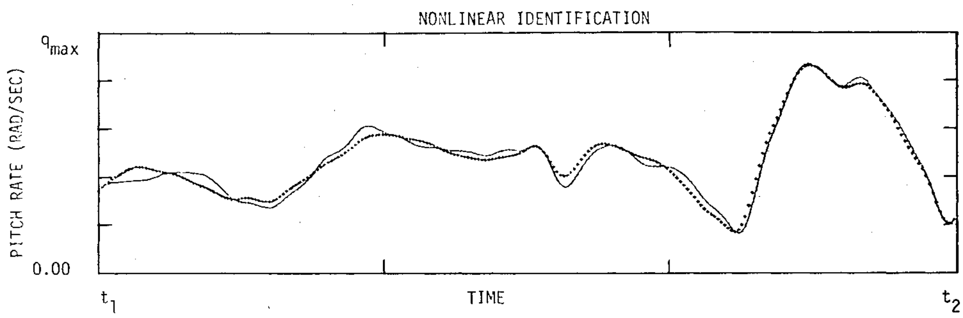

Fig. 8 Flight measured and identified model time histories for a MaRV.

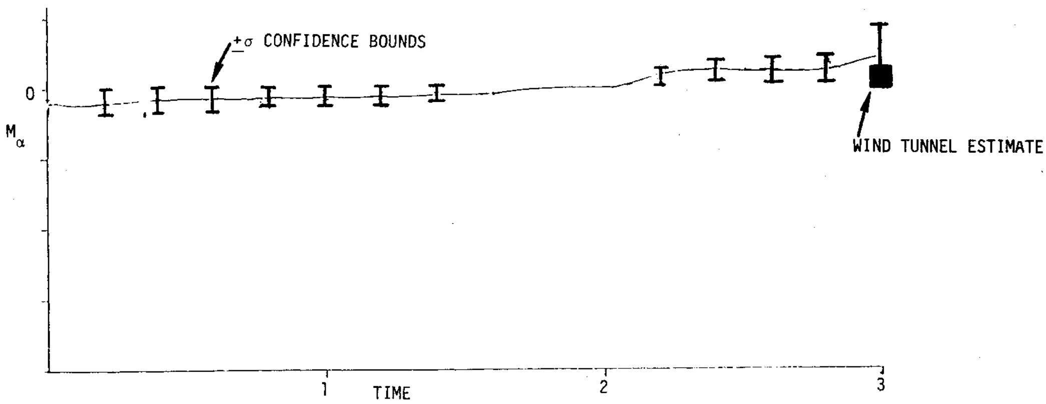

Fig. 9 Comparison of M_α estimates based on regression and on maximum likelihood.

Figure 5 shows the C_m vs. C_z characteristics used in the simulation at a specific Mach number. The principal independent variables at this Mach number are angle-of-attack and control input magnitude. A particular maneuver, of course, covers only a portion of this regime, as indicated by the shaded area (the Mach number in the maneuver is continuously varying).

The accuracy evaluation of the algorithm consists of the following:

1) The simulated data (noise free) is corrupted with instrumentation errors to create time histories which are then representative of actual flight data.

2) These time histories are then processed by the data reconstruction, model structure determination, and parameter identification algorithms to provide estimates of the C_m and C_z characteristics.

A result of such processing is shown in Fig. 6. The dashed lines indicate the identified C_m vs C_z characteristics. The identified variation of C_m and C_z at constant angle-of-attack is seen to be of slightly different slope than that of the simulation data base. This is due to the difficulty of extracting the dependence of C_z on the control deflection in the presence of on-board sensor errors. The agreement, nevertheless, is considered very good. It is significant to note, further, that the identified model predicts the aerodynamic characteristics of the vehicle with reasonable accuracy outside of the data window defined by Fig. 5.

Figure 7 is a typical plot of important aerodynamic stability and control parameters which are identified from the simulated, noise corrupted data. For both plots (C_{m_α} and C_{m_δ}) the time variation in the identified parameter is indicated with a solid line. The 2σ estimated confidence interval variation is shown by the bracketing curves. The identified values always fall with $\pm 2\sigma$ bounds of the true (i.e. simulation) values.

Evaluation of Algorithm Performance on Flight Data

The application of the algorithms to actual data validates the data processing architecture, and, to the extent that high-confidence wind tunnel data is available on regimes of the flight data, the accuracy of those algorithms.

The procedure for this evaluation is as follows:

1) Flight data is processed through the data reconstruction, model structure determination, and parameter identification algorithms.

2) Direct comparisons are made between flight data parameter estimates and wind tunnel estimates (if available).

Figure 8 shows the flight recorded pitch rate vs time for a segment of the reentry of an advanced maneuvering vehicle (solid line). The crosses represent the responses of the identified model. The maximum error between these two time histories is .02 rad/s. This fit is considered to be very good, given the known errors in the vehicle pitch rate sensor. Figure 9 shows the parameter estimates for C_m and from the identification algorithm. These results have the same trends as the wind tunnel data, but the maximum value of $C_{m\alpha}$ is significantly higher and the minimum value of C_{m_δ} significantly lower than wind tunnel data. The identified values successfully explained certain anomalous behavior observed in flight.

Summary

A general system identification approach has been specialized for the estimation of aerodynamic coefficients from maneuvering re-entry vehicle (MaRV) flight data. This specialization is necessary to extract the maximum amount of information from only a limited number of flights of such vehicles. The overall system identification approach consists of two main phases, the preflight system evaluation phase and the postflight data processing phase. The algorithms developed for the MaRV preflight system evaluation phase are 1) instrument analysis, 2) telemetry bandwidth evaluation, and 3) trajectory perturbation determination.

The steps developed for postflight data analysis are 1) data reconstruction, 2) model structure development, 3) maximum likelihood parameter identification, and 4) verification.

Preflight analysis and postflight data processing have been applied to both simulated and actual flight data. *Preflight* analysis results in specifications of sensor requirements and desired trajectory characteristics necessary for maximizing data quality for system identification accuracy. *Postflight* analyses quantifies significant aerodynamic and dynamic effects which are necessary for specifying performance of the vehicle and its guidance and control system.

Acknowledgments

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